

# SENSOR DATA CHARACTERISATION AND FUSION FOR TARGET TRACKING APPLICATIONS

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## ABSTRACT

Data Fusion based on data from several ground based target tracking radars, EOTs and INS sensors is a complex problem as the data contains different systematic errors, time stamps and time delays. This paper presents some practical solutions to correcting the sensor errors like bias, estimation of measurement and process noise, time stamp and time delay error handling. The solutions are implemented in a data fusion scheme for a tracking application. The data fusion scheme is tested on simulated data of a moving target and also applied to real data of an aircraft tracked by three ground based radars.

## INTRODUCTION

In surveillance applications, data from several sensors like ground based radars, electro optical transducers (EOT) and inertial navigation system (INS) are fused to obtain an accurate estimate of the position and velocity of the targets in the surveillance region. The problem of obtaining an optimal solution to the estimation of position and velocity of targets sensed by multiple sensors is complex, because the measurement data from the various sensors could have different errors, time stamps and delays. The data could contain both systematic as well as random errors. Random errors like process and measurement noise are handled by the estimation algorithm and generate an optimal solution. However, the systematic errors are not the same for all sensors and each of the sensor data has to be corrected separately before the data can be used for fusion. A way to correct systematic errors using an alignment algorithm was suggested in literature<sup>[1]</sup>, where the radar measurements are mapped to the ECEF coordinates using a geodetic transformation and the radar errors are estimated using least squares technique.

This paper aims at giving some practical solutions to the three commonly encountered problems in target tracking for surveillance applications namely, a) the estimation of systematic bias errors in sensors, b) adaptive estimation of measurement and process noise covariances and c) time stamp and time delay error handling. Finally, the corrected sensor data from the different measurement sensors are used for obtaining fused estimate of the position and velocity of the target.

In the following sections, descriptions of the techniques/algorithms used for sensor bias estimation, estimation of process and measurement noise covariances, time stamp error handling and data fusion are presented.

## ESTIMATION OF SENSOR BIAS ERRORS:

GPS data is used as reference to obtain estimate of sensor bias errors and the measurement noise covariance for the various sensors. The technique uses Kalman filter with an error state space formulation<sup>[2]</sup>. The error state space (also known as indirect) Kalman filter (ESKF) estimates the bias errors in the sensor data using the difference between the actual measured position data and the reference GPS data. The covariance of the residuals of the ESKF gives an estimate of the measurement noise covariance of the particular sensor. The estimated biases are used to correct the sensor data before it is used for state estimation and fusion. The block diagram of ESKF is shown in Figure 1.

## Data Transformation:

The sensor characterization is carried out in earth centered earth fixed (ECEF) coordinates. The GPS data in WGS-84 frame is first converted to ECEF frame :

$$\begin{aligned}
\mathbf{X}_{\text{eccf}} &= (\mathbf{r} + \mathbf{h}) \cos \lambda \cos \mu \\
\mathbf{Y}_{\text{eccf}} &= (\mathbf{r} + \mathbf{h}) \cos \lambda \sin \mu \\
\mathbf{Z}_{\text{eccf}} &= \{(1 - e^2)\mathbf{r} + \mathbf{h}\} \sin \lambda
\end{aligned} \tag{1}$$

where  $\lambda$ ,  $\mu$  and  $\mathbf{h}$  are longitude, latitude and altitude obtained from GPS

$\mathbf{r} = \mathbf{a}/(1 - e^2 \sin^2 \lambda)^{1/2}$  is effective radius of earth,

' $\mathbf{a}$ ' is the semi major axis (equatorial radius) = 6378.135 km and

' $\mathbf{e}$ ' is the eccentricity = 0.08181881.

The radar data measured in polar frame is converted to local cartesian coordinates in ENV (East North Vertical) frame :

$$\begin{aligned}
\mathbf{X}_{\text{env}} &= \mathbf{R} \sin \phi \cos \theta \\
\mathbf{Y}_{\text{env}} &= \mathbf{R} \cos \phi \cos \theta \\
\mathbf{Z}_{\text{env}} &= \mathbf{R} \sin \theta
\end{aligned} \tag{2}$$

where  $\mathbf{R}$  is range in meters,  $\phi$  is azimuth in degree and  $\theta$  is elevation in degree.

The sensor data is then transformed from ENV to ECEF frame as shown below.

$$\begin{bmatrix} \mathbf{X}_{\text{eccf}} \\ \mathbf{Y}_{\text{eccf}} \\ \mathbf{Z}_{\text{eccf}} \end{bmatrix} = \begin{bmatrix} \mathbf{X}_k \\ \mathbf{Y}_k \\ \mathbf{Z}_k \end{bmatrix} + \begin{bmatrix} -\sin \mu & -\sin \lambda \cos \mu & \cos \lambda \cos \mu \\ \cos \mu & -\sin \lambda \sin \mu & \cos \lambda \sin \mu \\ 0 & \cos \lambda & \sin \lambda \end{bmatrix} \begin{bmatrix} \mathbf{X}_{\text{env}} \\ \mathbf{Y}_{\text{env}} \\ \mathbf{Z}_{\text{env}} \end{bmatrix} \tag{3}$$

where  $\lambda$  and  $\mu$  are the latitude and longitude of the respective sensors.

$\mathbf{X}_k$ ,  $\mathbf{Y}_k$  and  $\mathbf{Z}_k$  give the location of the sensors in ECEF frame. This is obtained from the latitude ( $\lambda$ ), longitude ( $\mu$ ) and altitude ( $\mathbf{h}$ ) of the tracking station using eq. (1)

In case of INS, the measured down range, cross range and altitude is converted to local ENV frame and then to ECEF frame. Similarly, in case of EOTs, the measured azimuth and elevation are transformed to local ENV frame using least square algorithm and then the data is transformed to ECEF frame [3].

Also it is essential to time synchronize the GPS data and sensors data for estimating the bias errors.

#### Error State Space Kalman Filter (ESKF)<sup>[2]</sup>:

UD factorization based Kalman filter is used for estimating the sensor bias and measurement noise covariance. In sensor characterization, the Kalman filter is implemented with 'error state space' model (also known as indirect method) instead of actual state space model. The error model for characterizing the sensor is of the form:

$$\delta \mathbf{X}(\mathbf{k} + 1) = \Phi \delta \mathbf{X}(\mathbf{k}) + \mathbf{G} \mathbf{w}(\mathbf{k}) \tag{4}$$

$$\delta \mathbf{Z}(\mathbf{k}) = \mathbf{H} \delta \mathbf{X}(\mathbf{k}) + \mathbf{v}(\mathbf{k}) \tag{5}$$

where  $\delta \mathbf{X}$  vector of position and velocity error states in all the three axis

$\delta \mathbf{Z}$  vector of measured position error (i.e. GPS – Sensor) in all three axis

$\Phi$  transition matrix

$\mathbf{H}$  observation matrix

$\mathbf{w}$  process noise with mean zero and covariance  $\mathbf{Q}$

$\mathbf{v}$  measurement noise with mean zero and covariance  $\mathbf{R}$

The complete error model for characterizing the sensors in ECEF frame is given below:

$$\begin{bmatrix} \delta \mathbf{x}(k+1) \\ \delta \mathbf{v}_x(k+1) \\ \delta \mathbf{y}(k+1) \\ \delta \mathbf{v}_y(k+1) \\ \delta \mathbf{z}(k+1) \\ \delta \mathbf{v}_z(k+1) \end{bmatrix} = \begin{bmatrix} 1 & T & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & T & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \delta \mathbf{x}(k) \\ \delta \mathbf{v}_x(k) \\ \delta \mathbf{y}(k) \\ \delta \mathbf{v}_y(k) \\ \delta \mathbf{z}(k) \\ \delta \mathbf{v}_z(k) \end{bmatrix} + \begin{bmatrix} T^2/2 \\ T \\ T^2/2 \\ T \\ T^2/2 \\ T \end{bmatrix} \mathbf{w}(k) \quad (6)$$

$$\begin{bmatrix} \delta \mathbf{x}_m(k) \\ \delta \mathbf{y}_m(k) \\ \delta \mathbf{z}_m(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \delta \mathbf{x}(k) \\ \delta \mathbf{v}_x(k) \\ \delta \mathbf{y}(k) \\ \delta \mathbf{v}_y(k) \\ \delta \mathbf{z}(k) \\ \delta \mathbf{v}_z(k) \end{bmatrix} + \begin{bmatrix} \mathbf{v}_1(k) \\ \mathbf{v}_2(k) \\ \mathbf{v}_3(k) \end{bmatrix} \quad (7)$$

where  $\delta \mathbf{x}$  = position error in X-direction

$\delta \mathbf{v}_x$  = velocity error in X-direction

$\delta \mathbf{y}$  = position error in Y-direction

$\delta \mathbf{v}_y$  = velocity error in Y-direction

$\delta \mathbf{z}$  = position error in Z-direction

$\delta \mathbf{v}_z$  = velocity error in Z-direction

$T$  = sampling time

subscript  $m$  represents measured position error (GPS – SENSOR)

### ESTIMATION OF MEASUREMENT AND PROCESS NOISE COVARIANCE:

In addition to correcting the bias, which is a systematic error, for achieving optimal fusion an estimate of the measurement noise covariance ( $R$ ) and process noise covariance ( $Q$ ) is required. A sliding window technique is used for adaptive estimation of  $R$  in each of the measurement channels. An estimate of  $R$  is obtained by finding the covariance of the residuals (from ESKF) over a chosen window length. Using the estimated  $R$ , the  $Q$  is adaptively estimated during the state estimation using the method given in reference<sup>[2]</sup> which is shown below.

$$\sum_{k=i-N+1}^i [\Phi P(k-1/k-1) \Phi^T + G Q(k-1) G^T - P(k/k) - \Delta \mathbf{x}(k) \Delta \mathbf{x}(k)^T] = 0 \quad (8)$$

where

$$\Delta \mathbf{x}(k) = \hat{\mathbf{x}}(k/k) - \hat{\mathbf{x}}(k/k-1) = \mathbf{K}(k) \mathbf{r}(k) \quad (9)$$

$$\mathbf{P}(k/k) = \mathbf{P}(k-1/k-1) - \mathbf{K}(k) \mathbf{H} \mathbf{P}(k-1/k-1) \quad (10)$$

$$\mathbf{P}(k-1/k-1) = \mathbf{K}(k) \hat{\mathbf{A}}(k) \mathbf{H}^T \quad (11)$$

$$\hat{\mathbf{A}}(k) = \frac{1}{N} \sum_{k=i-N+1}^i \mathbf{r}(k) \mathbf{r}(k)^T \quad (12)$$

If  $G$  is invertible for all  $k$ , then an estimate of  $Q(k)$  can be defined as

$$\hat{\mathbf{Q}}(i) = \frac{1}{N} \sum_{k=i-N+1}^i \left\{ \mathbf{G}^{-1} \left[ \Delta \mathbf{x}(k) \Delta \mathbf{x}(k)^T + \mathbf{P}(k/k) - \Phi \mathbf{P}(k-1/k-1) \Phi^T \right] \mathbf{G}^{-T} \right\} \quad (13)$$

If  $\mathbf{G}$  is not invertible, then pseudo inverse, computed as

$$\mathbf{G}^\# = \left[ \mathbf{G}^T \mathbf{G} \right]^{-1} \mathbf{G}^T \quad (14)$$

can be used.

#### TIME STAMP AND TIME DELAY ERROR HANDLING:

While fusing data from two or more sensors, it is essential that the measurements from the sensors are available at the same instant of time and that the data is received with an accurate time stamp corresponding to the time at which the data is sensed/acquired by the sensor. However, this may not be the case in practice, where data could come with erroneous time stamps either at the transmitting end or at the receiving end or there could be a drift in the time recorded on any channel. The time drift could either be a constant value on any channel or the time drift could be different (random) at each instant of time. Also, the measurement samples in different radars may be different.

Several practical solutions to this issue have been studied: (i) the first few samples of data are used to ascertain constant time drift on any channel and the subsequent time stamps are corrected by the computed value of time drift and (ii) in order to handle random time drift, at any instant of time, any data coming within one half of the sampling time on that channel is treated as if it has arrived at that instant for purpose of fusion (iii) for state vector fusion, it is expected that the output of the filter is available every  $T_s$  secs (reference sampling time). This is achieved by using the data arriving at each instant for updating the states of the filter and propagating the estimated states to the nearest  $T_s$  so that it is available for fusion there. Of course, this method of synchronizing the state vectors for fusion presupposes that the sampling time requirements are appropriately chosen to suit the dynamics of the target being tracked.

#### MULTI SENSOR DATA FUSION SCHEME:

The Figure 2 shows the block diagram of the adopted multi sensor data fusion scheme.

The first step in this data fusion scheme is time synchronization of various sensors data. For the purpose of time synchronization, the data arrival on each of the tracking sensor data (including GPS) is checked and a reference time signal is initiated using the time stamp on the sensor data that arrives first. The reference time signal is incremented at a uniform rate of  $T_s$  secs. At each  $T_s$  sec, the sensor time stamp on each of the input data channels is compared with the reference time and appropriate action is initiated using 'decision logic' based on time delay error handling procedure mentioned above.

Each sensor data is appropriately transformed to ECCF frame using standard transformation equations mentioned earlier. And then with GPS signal as reference, all the sensors data are characterized by estimating bias and measurement and process noise covariance as mentioned previously. This preprocessed data is then used for filtering and fusion as described below with models and governing equations.

#### Tracking Model:

The target motion is modeled as

$$\mathbf{X}(k+1) = \Phi \mathbf{X}(k) + \mathbf{G} \mathbf{w}(k) \quad (15)$$

where

$\mathbf{X}$  is state vector consisting of target position, velocity and acceleration in all the three axes:

$$\mathbf{X} = [\mathbf{x}_p \quad \mathbf{x}_v \quad \mathbf{x}_a \quad \mathbf{y}_p \quad \mathbf{y}_v \quad \mathbf{y}_a \quad \mathbf{z}_p \quad \mathbf{z}_v \quad \mathbf{z}_a]^T$$

$\Phi$  is state transition matrix,  $\Phi =$

$$\begin{bmatrix} 1 & T & T^2/2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & T & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & T & T^2/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & T & T^2/2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$\mathbf{G}$  is matrix associated with process noise,  $\mathbf{G} =$

$$\begin{bmatrix} T^3/6 & 0 & 0 \\ T^2/2 & 0 & 0 \\ T & 0 & 0 \\ 0 & T^3/6 & 0 \\ 0 & T^2/2 & 0 \\ 0 & T & 0 \\ 0 & 0 & T^3/6 \\ 0 & 0 & T^2/2 \\ 0 & 0 & T \end{bmatrix}$$

$\mathbf{w}$  is process noise with  $\mathbf{E}[\mathbf{w}(k)] = \mathbf{0}$  and  $\text{cov}[\mathbf{w}(k)] = \mathbf{Q}$

$$\mathbf{Z}(k) = \mathbf{H} \mathbf{X}(k) + \mathbf{v}(k) \quad (16)$$

where

$\mathbf{Z}$  is measurement vector given by  $\mathbf{Z} = [\mathbf{x}_p \quad \mathbf{y}_p \quad \mathbf{z}_p]^T$

$\mathbf{H}$  is observation matrix given by  $\mathbf{H} =$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$\mathbf{v}$  is measurement noise with  $\mathbf{E}[\mathbf{v}(k)] = \mathbf{0}$  and  $\text{cov}[\mathbf{v}(k)] = \mathbf{R}$

#### **Kalman Filter for Trajectory Estimation:**

Kalman filter is implemented in UD factorization form<sup>[4]</sup> for the trajectory estimation using target motion model mentioned above. The states and state error covariance are estimated as follows:

#### **U-D factor Time Propagation**

State estimate extrapolation :

$$\tilde{\mathbf{x}}(k+1) = \Phi \hat{\mathbf{x}}(k) \quad (17)$$

Error covariance extrapolation :

$$\tilde{\mathbf{P}}(k+1) = \Phi \hat{\mathbf{P}}(k) \Phi^T + \mathbf{G} \mathbf{Q} \mathbf{G}^T \quad (18)$$

Given  $\hat{\mathbf{P}} = \hat{\mathbf{U}}\hat{\mathbf{D}}\hat{\mathbf{U}}^T$  and  $\mathbf{Q}$  as the process noise covariance matrix, the time update factors  $\tilde{\mathbf{U}}$  and  $\tilde{\mathbf{D}}$  are obtained through modified Gram-Schmidt orthogonalization process.

We may define  $\mathbf{W} = [\Phi\hat{\mathbf{U}} | \mathbf{G}_A] \bar{\mathbf{D}} = \text{diag}[\hat{\mathbf{D}}, \mathbf{Q}]$  with  $\mathbf{W}^T = [\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_n]$

where  $\mathbf{P}$  is reformulated as  $\tilde{\mathbf{P}} = \tilde{\mathbf{W}}\tilde{\mathbf{D}}\tilde{\mathbf{W}}^T$ . The  $\mathbf{U}$  and  $\mathbf{D}$  factors of  $\tilde{\mathbf{W}}\tilde{\mathbf{D}}\tilde{\mathbf{W}}^T$  may be computed as described below.

For  $j=n, n-1, \dots, 2$ , the following equations are recursively evaluated.

$$\begin{aligned} \tilde{\mathbf{D}}_j &= \langle \mathbf{w}_j^{(n-j)}, \mathbf{w}_j^{(n-j)} \rangle_D \\ \tilde{\mathbf{U}}(i, j) &= \langle \mathbf{w}_i^{(n-j)}, \mathbf{w}_j^{(n-j)} \rangle_D / \tilde{\mathbf{D}}_j \quad i=1, \dots, (j-1) \\ \mathbf{w}_i^{(n-j+1)} &= \mathbf{w}_i^{(n-j)} - \tilde{\mathbf{U}}(i, j)\mathbf{w}_j^{(n-j)} \quad i=1, \dots, (j-1) \\ \tilde{\mathbf{D}}_1 &= \langle \mathbf{w}_1^{(n-1)}, \mathbf{w}_1^{(n-1)} \rangle_D \end{aligned} \quad (19)$$

Here subscript  $D$  is used to denote the weighted inner product w.r.t.  $D$ .

### U-D Factor Measurement Update

The measurement update in Kalman filtering combines a priori estimate  $\tilde{\mathbf{x}}$  and error covariance  $\tilde{\mathbf{P}}$  with scalar observation  $\mathbf{z} = \mathbf{a}^T \mathbf{x} + \mathbf{v}$  to construct an updated (filtered state) estimate and covariance:

$$\begin{aligned} \mathbf{K} &= \tilde{\mathbf{P}}\mathbf{a} / \alpha \\ \hat{\mathbf{x}} &= \tilde{\mathbf{x}} + \mathbf{K}(\mathbf{z} - \mathbf{a}^T \tilde{\mathbf{x}}) \\ \alpha &= \mathbf{a}^T \tilde{\mathbf{P}}\mathbf{a} + r \\ \hat{\mathbf{P}} &= \tilde{\mathbf{P}} - \mathbf{K}\mathbf{a}\tilde{\mathbf{P}} \end{aligned} \quad (20)$$

where  $\tilde{\mathbf{P}} = \tilde{\mathbf{U}}\tilde{\mathbf{D}}\tilde{\mathbf{U}}^T$ , ' $\mathbf{a}$ ' is the measurement vector/matrix, ' $r$ ' is the measurement noise covariance and  $\mathbf{z}$  is the string of noisy measurements.

Kalman gain  $\mathbf{K}$  and updated covariance factors  $\hat{\mathbf{U}}$  and  $\hat{\mathbf{D}}$  can be obtained from the following equations:

$$\begin{aligned} \mathbf{f} &= \tilde{\mathbf{U}}^T \mathbf{a}; \quad \mathbf{f}^T = (\mathbf{f}_1, \dots, \mathbf{f}_n) \\ \mathbf{v} &= \tilde{\mathbf{D}}\mathbf{f}; \quad \mathbf{v}_i = \tilde{\mathbf{d}}_i \mathbf{f}_i \quad i=1, 2, \dots, n \\ \hat{\mathbf{d}}_1 &= \tilde{\mathbf{d}}_1 r / \alpha_1; \quad \alpha_1 = r + \mathbf{v}_1 \mathbf{f}_1; \quad \mathbf{K}_2^T = (\mathbf{v}_1 \ 0 \dots 0) \end{aligned} \quad (21)$$

For  $j=2, \dots, n$  recursively the following equations are evaluated:

$$\begin{aligned} \alpha_j &= \alpha_{j-1} + \mathbf{v}_j \mathbf{f}_j \\ \hat{\mathbf{d}}_j &= \tilde{\mathbf{d}}_j \alpha_{j-1} / \alpha_j \\ \hat{\mathbf{u}}_j &= \tilde{\mathbf{u}}_j + \lambda_j \mathbf{k}_j; \quad \lambda_j = -\mathbf{f}_j / \alpha_{j-1} \\ \mathbf{K}_{j+1} &= \mathbf{K}_j + \mathbf{v}_j \tilde{\mathbf{u}}_j \end{aligned} \quad (22)$$

where  $\tilde{\mathbf{U}} = [\tilde{\mathbf{u}}_1, \dots, \tilde{\mathbf{u}}_n]$ ,  $\hat{\mathbf{U}} = [\hat{\mathbf{u}}_1, \dots, \hat{\mathbf{u}}_n]$  and Kalman gain is given by  $\mathbf{K} = \mathbf{K}_{n+1} / \alpha_n$  where  $\tilde{\mathbf{d}}$  is predicted diagonal element and  $\hat{\mathbf{d}}_j$  is the updated diagonal element of the  $D$  matrix.

### **Measurement Fusion<sup>[5]</sup>:**

The trajectory data from similar radars like S band radars (GR1, GR2, GR3) are fused using measurement fusion. In this approach the algorithm fuses the sensor observations directly and uses a Kalman filter to estimate the fused state vector. The equations describing this process are same as

equations 17 to 21 except that the observation matrix is augmented taking all the sensor outputs

together as  $\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ . \\ . \\ \mathbf{H}_n \end{bmatrix}$

where,  $\mathbf{H}_1, \mathbf{H}_2, \dots, \mathbf{H}_n$  are observation matrix of  $n$  individual sensors (as shown in equation 16).

Similarly, the measurement covariance matrix  $\mathbf{R} = \begin{bmatrix} \mathbf{R}_1 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{R}_2 & 0 & 0 & 0 \\ 0 & 0 & . & 0 & 0 \\ 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & 0 & \mathbf{R}_n \end{bmatrix}$

where  $\mathbf{R}_1, \mathbf{R}_2, \dots, \mathbf{R}_n$  are the measurement error covariance of respective sensors.

### State Vector Fusion<sup>[6]</sup>

The four trajectories each from the individual modules (Figure 2) are fused using state vector fusion. The fusion is performed as follows:

$$\hat{\mathbf{X}}^f = \hat{\mathbf{X}}^1 + \hat{\mathbf{P}}^1(\hat{\mathbf{P}}^1 + \hat{\mathbf{P}}^2)^{-1}(\hat{\mathbf{X}}^2 - \hat{\mathbf{X}}^1) \quad (23)$$

$$\hat{\mathbf{P}}^f = \hat{\mathbf{P}}^1 - \hat{\mathbf{P}}^1(\hat{\mathbf{P}}^1 + \hat{\mathbf{P}}^2)^{-1}\hat{\mathbf{P}}^{1T} \quad (24)$$

where  $\hat{\mathbf{X}}^1$  and  $\hat{\mathbf{X}}^2$  are the state vector of estimated trajectory 1 and 2, and  $\hat{\mathbf{P}}^1$  and  $\hat{\mathbf{P}}^2$  are the state error covariance of estimated trajectory 1 and 2.

In this method only two trajectories can be fused at a time.

### Fusion Philosophy

For the purpose of fusion, all the tracking sensors are grouped (Figure 2) into four major groups based on the type, sensitivity and accuracy of the sensors.

(i) Module 1: The tracking data from the three S band radars (GR1, GR2, GR3) after coordinate transformation and characterization are fused to get single trajectory data (**trajectory 1**) by direct measurement fusion using UD factorization based Kalman filter.

(ii) Module 2: The tracking data from the PCMC radar after coordinate transformation and characterization is filtered (**trajectory 2**) using UD factorization Kalman filter.

(iii) Module 3: The tracking data (azimuth and elevation) from EOTs are first transformed to local ENV (east north vertical) frame using least square method and then transformed to ECEF. This data is then filtered (**trajectory 3**) using UD factorization based Kalman filter.

(iv) Module 4: The tracking data from the onboard inertial navigation system (INS) is telemetered at sampling interval of 72 msec. This data (down range, cross range and altitude) is transformed to ECEF, characterized and filtered (**trajectory 4**). The data is then time propagated in ECEF coordinates using target dynamic model to time synchronize with other track data.

These four trajectories (trajectory 1, trajectory 2, trajectory 3 and trajectory 4) are then fused using state vector fusion. As mentioned earlier, since state vector fusion algorithm permits fusion of two trajectories at a time, a hierarchical order based on the accuracies of sensors is chosen for generating the final trajectory estimates. Since EOT sensors are accurate upto a maximum range of 40 kms, this is used to decide the priority for fusion as given below:

If estimated range is less than 40 km, the trajectory 3 (EOT data) is fused with trajectory 1 (S band radars). The resultant trajectory is fused with trajectory 2 (PCMC) and finally this resultant trajectory is fused with trajectory 4 (INS). And if the estimated range is greater than 40 km, the trajectory 1 is fused with trajectory 2 and the resultant trajectory is finally fused with trajectory 4. That is, if the estimated range is less than 40 km, the trajectory 3 (EOT data) is used for fusion but for ranges beyond 40km the trajectory 3 is not included for state vector fusion.

## RESULTS AND DISCUSSIONS

The sensor characterization, trajectory filtering and fusion techniques/algorithms described above have been developed in 'C' on UNIX platform. The entire scheme is validated with the simulated trajectory data of all the sensors, generated using a GUI based 'simulator program' which generates noisy measurement data of a moving target launched from a given location. It generates measurements of i)  $R, \theta, \phi$  for **S** and **C** band radars, ii) cross range, down range, and altitude for **INS**, iii)  $\theta, \phi$  for **EOTs**, and iv) WGS-84 for **GPS**.

The results are presented in terms of:

1. Mean of residuals.
2. Percentage autocorrelation values out of the  $2\sigma$  theoretical error bounds.
3. Percentage innovation values out of bounds.
4. Percentage fit error w.r.t true states.

And also by plotting the following:

1. Estimated position with GPS data.
2. Innovation sequence with  $2\sigma$  bounds.
3. Auto correlation of residuals with bounds.
4. Root sum squares of position error (RSSPE).
5. State error covariance (Trace of error covariance matrix P).

The table 1 shows the (a) mean of residuals, (b) percentage autocorrelation values out of bounds, (c) percentage innovation values out of bounds and (d) percentage fit error w.r.t true states, obtained at different levels of fusion.

Figure 3 shows the plot of (a) estimated position with GPS data, (b) innovation sequence with  $2\sigma$  bounds, (c) auto-correlation of residuals with bounds (obtained at fusion level A). It can be seen from the figure that the performance of the tracking filters is satisfactory in terms of innovations and autocorrelations being within their theoretical bounds. Figure 4 shows the comparison of root sum square position error (RSSPE) and also the comparison of state error covariance obtained at different levels of fusion. From this figure it is demonstrated that the error in the final fused trajectory is minimum and that there is a decrease in the error covariance (because of increase in information) by fusing multiple trajectories.

Table 2 gives the results of sensor characterization of two S band and one PCMC radar from the real data (generated by flying an aircraft for purposes of sensor characterization and fusion) of an aircraft tracked by these ground based radars. Figure 5 shows the comparison of RSSPE and also the state error covariance after the fusion of these three real (radar) data. From the figure it is clear that the RSSPE is less than 40 meters for this application.

## CONCLUSIONS

Data fusion based on data from several ground-based radars, EOTs and INS channels is a complex problem because the data contains different errors, time stamps and time delays. Some practical solutions to correcting the sensor errors, estimating the noise covariance online and handling the asynchronous sensors are presented. The solutions are implemented in a data fusion scheme for a tracking application. The data fusion scheme is tested on simulated and real data. The results indicate a satisfactory performance of the proposed solutions in generating final state estimates with low errors.



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Table 1: Performance evaluation results of data fusion from simulated data

Level <sup>*</sup> →		A (S Band Radars)	B (C Band Radars)	C (EOTs)	D (INS)	E (Fused)
Mean of Residuals	X	0.00384	0.012823	0.000088	-0.01219	-----
	Y	-0.00432	-0.01401	-0.00165	0.006237	-----
	Z	0.00039	-0.00091	-0.00014	0.001789	-----
Percentage autocorrelation values out of bounds	X	3.640672	3.296703	2.973497	2.196855	-----
	Y	4.588539	5.214393	4.029304	2.218393	-----
	Z	2.951314	3.102780	3.210515	7.452078	-----
Percentage innovation values out of bounds	X	1.895735	1.659125	1.659125	2.326082	-----
	Y	0.366221	1.594484	0.193924	2.175318	-----
	Z	0.452391	0.667959	0.646412	0.387680	-----
Percentage fit error w.r.t true data	X	<b>0.098634</b>	<b>0.198129</b>	<b>0.012437</b>	<b>0.112249</b>	<b>0.101046</b>
	Y	<b>0.005032</b>	<b>0.009710</b>	<b>0.000702</b>	<b>0.004352</b>	<b>0.005161</b>
	Z	<b>0.001987</b>	<b>0.002518</b>	<b>0.000727</b>	<b>0.001011</b>	<b>0.001314</b>

\* A,B,C,D,E are at different levels in data fusion scheme as indicated in figure 2.

Table 2: Sensor characterization results from real data.

	X Position		Y Position		Z Position	
	Bias (m)	Meas Cov (m <sup>2</sup> )	Bias (m)	Meas Cov (m <sup>2</sup> )	Bias (m)	Meas Cov (m <sup>2</sup> )
GR1	-12.9	4100	-133.2	6400	155.6	3400
GR 2	-116.8	424	-136.9	8600	-409.1	1100
PCMC	241.5	896	-292.7	1300	137.5	1400

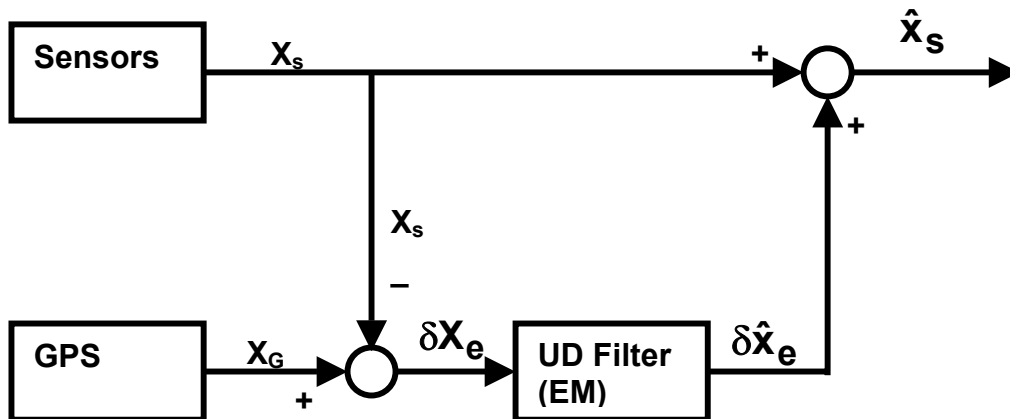


Figure 1: Kalman filter in sensor characterization mode

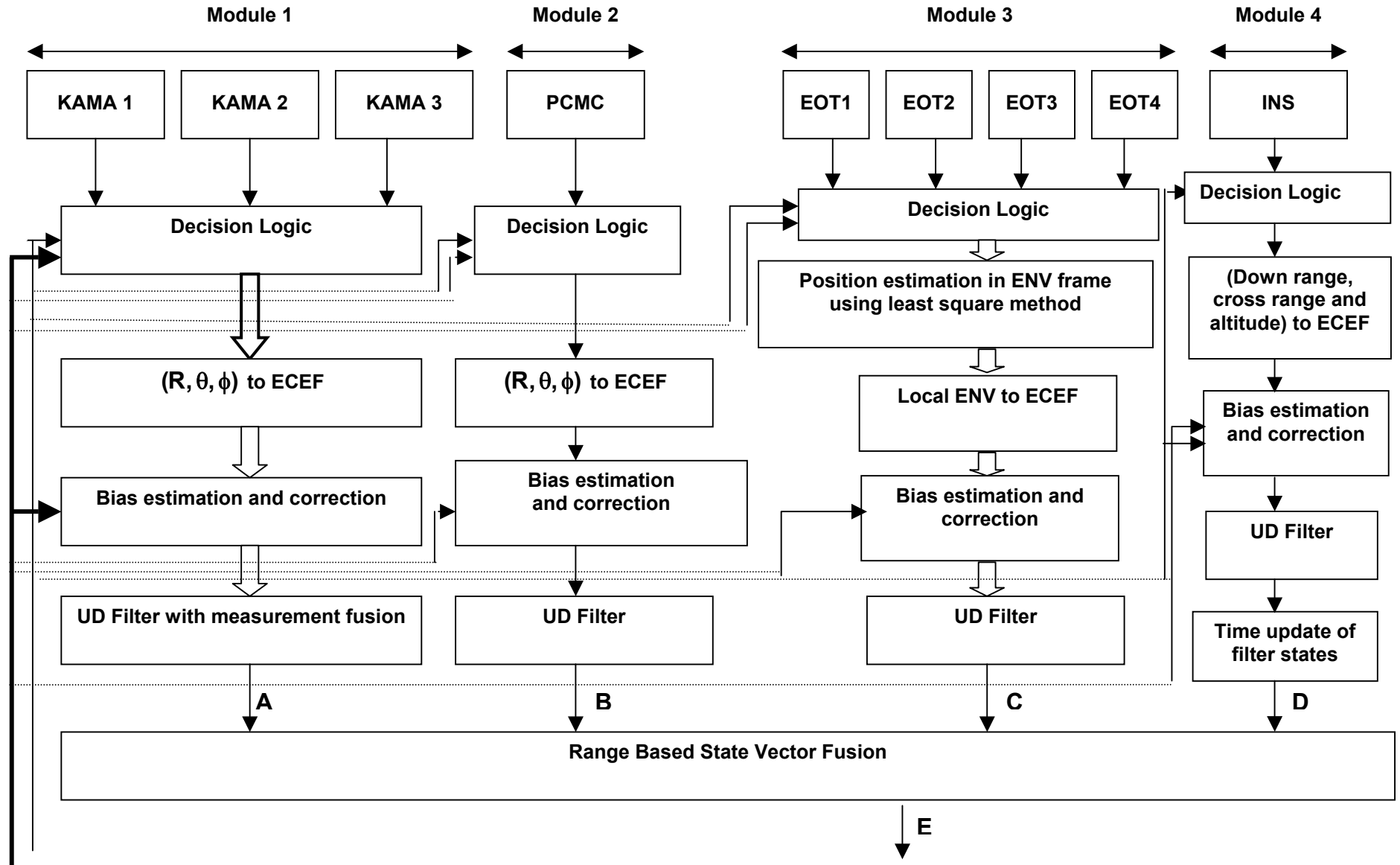


Figure 2: Block diagram of the data fusion scheme

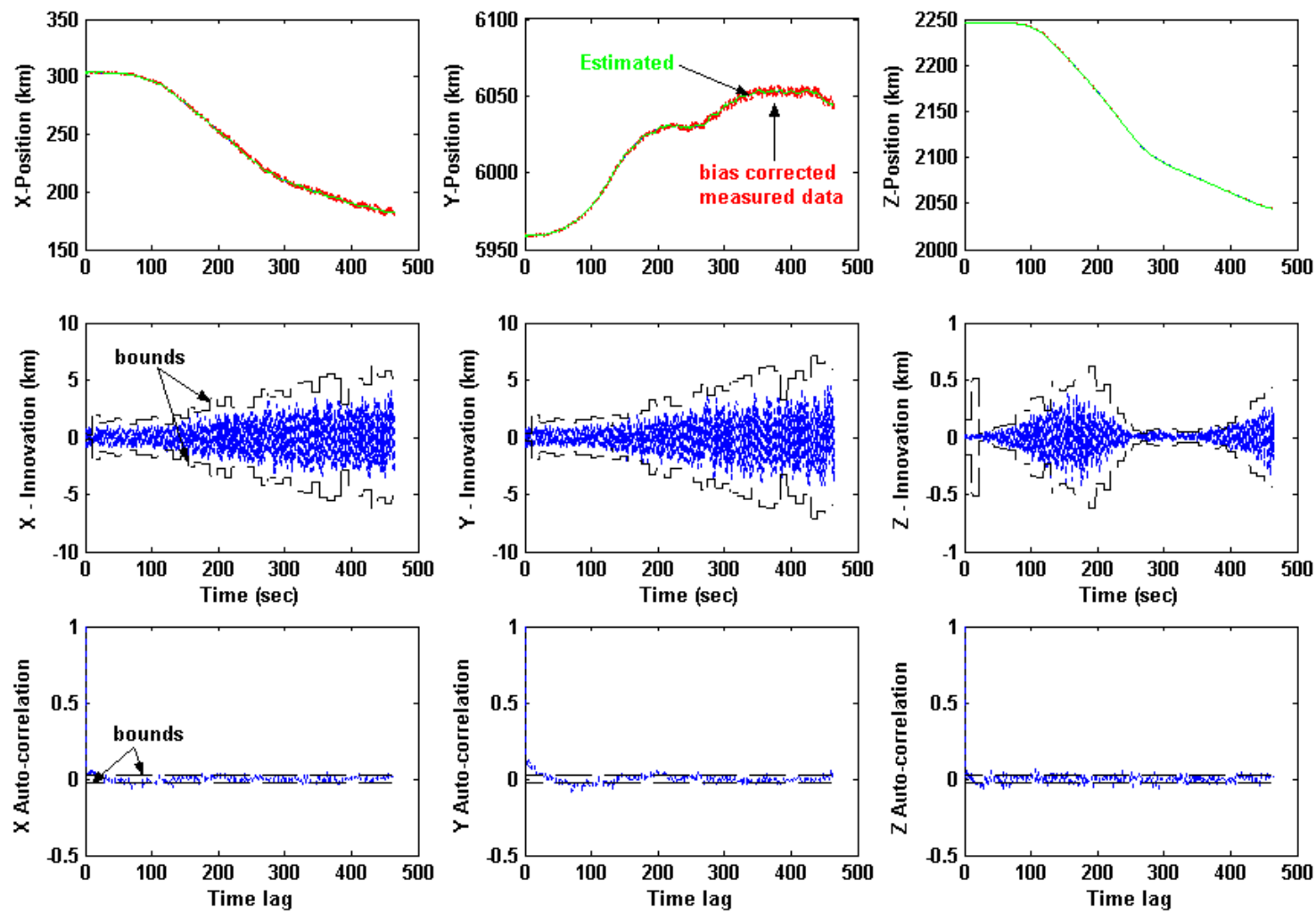


Figure 3: Performance evaluation of fusion filter at data fusion level A (refer figure 2)

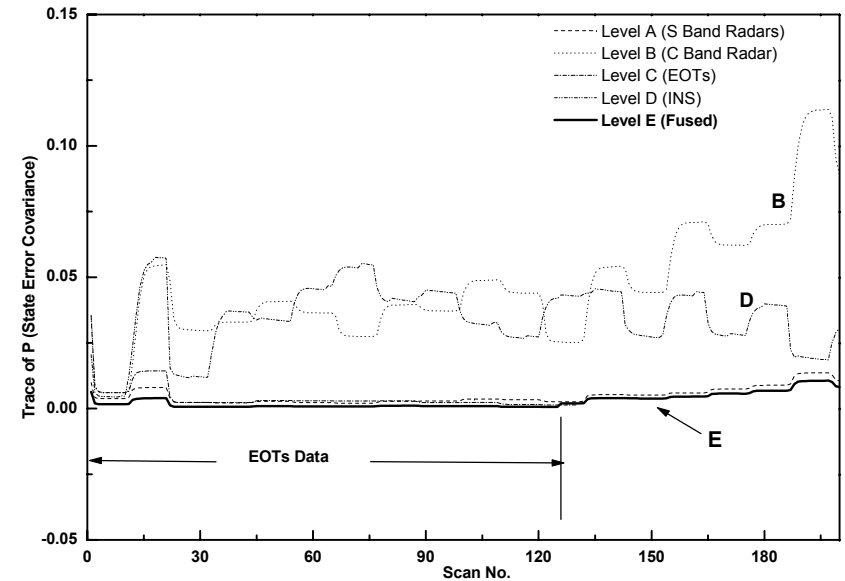
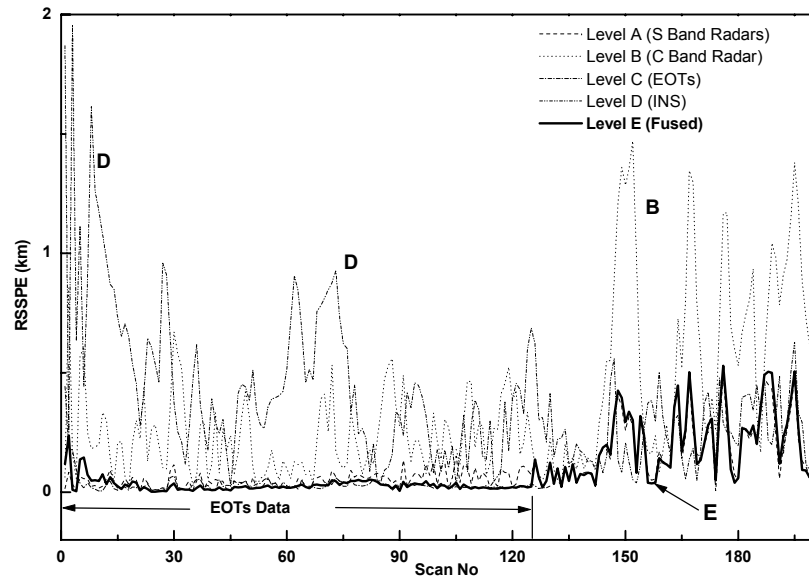


Figure 4: Performance evaluation of fusion scheme for simulated data

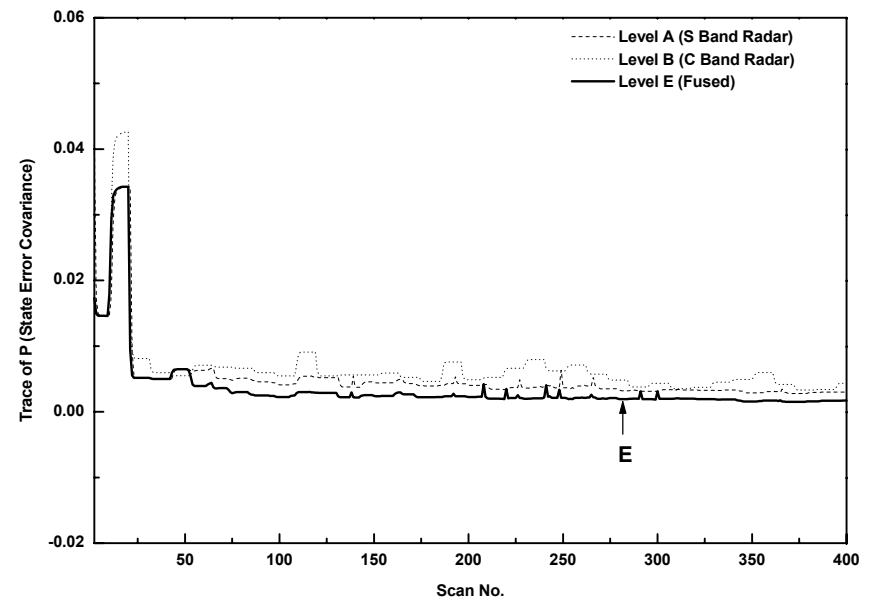
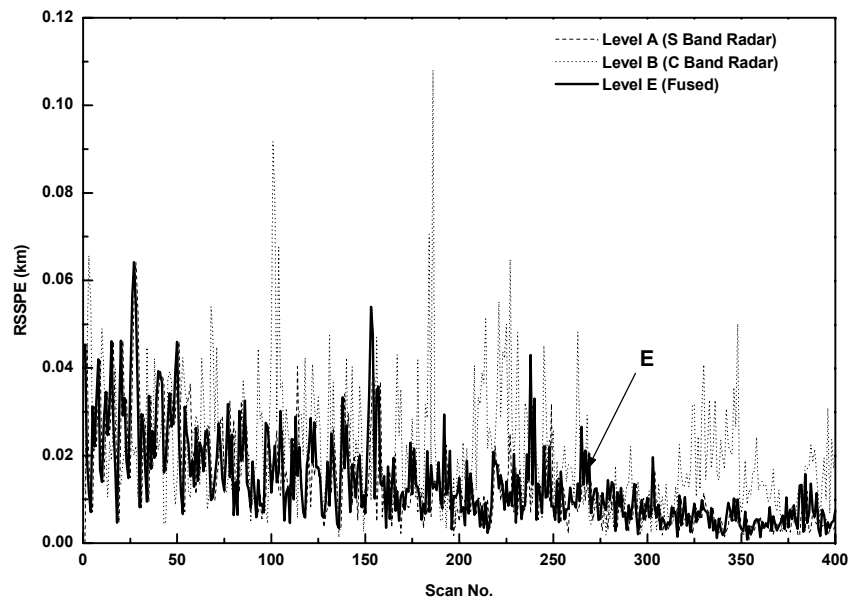


Figure 5: Performance evaluation of fusion scheme for real data

## **Bio Data of Authors**



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